Optimization & Linear Programming

FORENSIC ACCOUNTANTS

Agenda

> Introduction to Optimisation and Linear Programming
> Industry Examples
> Insurance Claim Examples
> Power Generation Worked Example
> Conclusions

Introduction

- Optimisation & linear programming are subcomponent of Operations Research and Management Science
- OR / MS definition:
  - *"…a discipline that deals with the application of advanced analytical methods to help make better decisions…"*
- History:
  - Discipline developed during World War I and II
  - Petroleum industry was an early adopter
- OR / MS wide number of applications:
  - Transportation / Logistics
  - Manufacturing / Product Mix
  - Pricing / Revenue maximisation

Sample Problem

> 500 square acre farm, can grow:
  - Wheat
  - Barley
  - Corn
  - Combination of the 3

> Limited supply of fertiliser and pesticide, both of which are needed (in different quantities) for each crop grown.

> So, how much of each crop should you grow to maximize your profit?

Characteristics of the Problem

> Decision:
- How much of each crop should be grown?
- What price to charge per product?

> Constraints:
- Expressed as greater than, less than or equal to
- Limited capacity of farm size
- Limited supply and quantity of fertiliser and pesticide
- Limited amount of labour and materials

> Objective:
- Maximise profit
- Minimise cost

Problem Formulation

1. Understand the problem
2. Identify the decision variables
3. State the objective function as a linear combination of the decision variables
4. State the constraints as a linear combination of the decision variables
5. Identify upper / lower bounds on the decision variables
6. Solve!

Solving the Problem

> Intuitive Approach:
- i.e. guess and test

> Graphical Approach:
- Plot the constraints
- Identify the feasible region
- Plot the objective function

> Mathematically:
- LP Model

Industry Examples

Product Blending

- Number of components or commodities can be mixed together to yield one or more products.
- Typically, different components / commodities are purchased. Each commodity has known characteristics and costs.
- LP can determine quantity of each commodity to purchase and blend with other products so that product specifications are met and the total cost is minimized.

Production Mix

- A manufacturer has fixed amounts of different resources such as raw material, labor, and equipment.
- These resources can be combined to produce any one of several different products.
- The quantity of the resource required to produce one unit of the each product is known.
- LP used to define the combination of products that will maximize total revenue

Transportation

- Company has several factories, and several suppliers.

- The cost of shipping a unit from its origin to the destination is known for all combinations of origins and destinations.

- LP used to determine the amount to be shipped from each origin to each destination such that the total cost of transportation is a minimum.

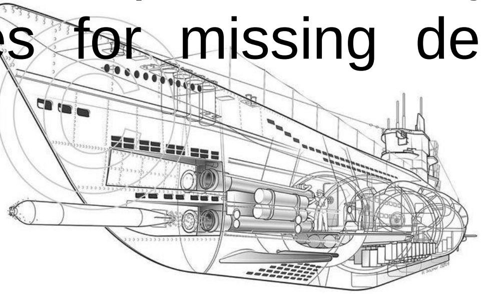
Logistics

- Manufacturer must supply a given number of items of a certain product each month for the next number of months.
- Can be produced in regular time, subject to a maximum each month, or in overtime. The cost of producing an item during overtime is greater than during regular time. A storage cost is associated with each item not sold at the end of the month.
- LP used to determine the production schedule that minimizes the sum of production and storage costs.

Military Logistics

- US Department of Defense have a logistics planning problem that models the feasibility of supporting military operations during a crisis.

- LP includes capacities at embarkation and debarkation ports, capacities of the various aircraft and ships that carry the movement requirements and penalties for missing delivery dates.



- This resulted in an LP with 20,500 constraints and 520,000 variables. Takes 75 minutes to solve.

Airline Crew Scheduling

- Number of different flight legs
- Aircraft working limitations (not all crew can work on all aircraft types)
- Working conditions restrictions
- 12 million potential crew schedules
- LP constraints used to ensure that all flight legs have a crew assigned to them, and work restrictions are violated.

Insurance Claim Examples

Examples

> Aviation:

- Optimisation of revenue that would have been earned by an airline had a key airport not been closed following interruption.

> Oil and Gas:

- Optimisation of crudes consumption and refinery operation following an outage

> Power Generation:

- Optimisation of portfolio of assets following an outage.

> Transportation:

- Optimisation of fleet of trains / routes to be used following the loss of a train.

Power Generation Worked Example

So2 Emissions Reduction

> Outage at 1,468.5 MW Coal fired station

> Date of Outage June 2008

> Unit returned to service in April 2009

> Part of claim included So2 credits – allowance could be transferred to other units owned by the Insured.

> Incentive existed to generate as much as possible with coal fired stations to ensure that the SO2 allocation was fully utilized but not exceeded.

Claim

- Claimed SO_2 credit calculation structured around:
  - average efficiency rates
  - monthly distribution of So2 emissions
  - between the three remaining coal fired stations
  - based on the actual generation / emission levels.

- Saved (as a consequence of the loss) So2 emissions were allocated to all the remaining plants based
  - Established monthly distributions
  - the relevant efficiency rates and spark spreads

- Total claimed calculated credit EUR 6.2 million

Claim – Transfer of So2

Description / Location	Unit	Jun-08	Jul-08	Aug-08	Sep-08	Oct-08	Nov-08	Dec-08	Total
Generation:									
Alternative Location 1	*Mwh*	182,728	644,506	530,705	476,818	530,798	559,197	511,981	3,436,732
Alternative Location 2	*Mwh*	8,973	237,147	94,188	112,367	309,631	304,619	317,621	1,384,546
Alternative Location 3	*Mwh*	168,743	325,951	163,470	312,825	428,331	250,312	490,111	2,139,744
Total	***Mwh***	**360,444**	**1,207,604**	**788,363**	**902,009**	**1,268,760**	**1,114,128**	**1,319,714**	**6,961,022**
So2 Emissions:									
Alternative Location 1	*Tonnes*	619	2,926	814	917	1,051	890	866	8,083
Alternative Location 2	*Tonnes*	32	669	260	332	706	882	919	3,799
Alternative Location 3	*Tonnes*	540	1,315	647	1,034	1,352	926	1,889	7,703
Total	***Tonnes***	**1,191**	**4,910**	**1,721**	**2,284**	**3,109**	**2,698**	**3,674**	**19,586**
Monthly Distribution of So2 Emissions:									
Alternative Location 1	*%*	51.96%	59.60%	47.28%	40.16%	33.81%	32.99%	23.57%	41.27%
Alternative Location 2	*%*	2.66%	13.62%	15.10%	14.55%	22.71%	32.68%	25.02%	19.40%
Alternative Location 3	*%*	45.38%	26.78%	37.61%	45.29%	43.47%	34.34%	51.41%	39.33%
Total	***%***	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**
So2 Emissions v Generation:									
Alternative Location 1	*Tonnes / Mwh*	0.0034	0.0045	0.0015	0.0019	0.0020	0.0016	0.0017	
Alternative Location 2	*Tonnes / Mwh*	0.0035	0.0028	0.0028	0.0030	0.0023	0.0029	0.0029	
Alternative Location 3	*Tonnes / Mwh*	0.0032	0.0040	0.0040	0.0033	0.0032	0.0037	0.0039	
Total	***Tonnes / Mwh***	**0.0033**	**0.0041**	**0.0022**	**0.0025**	**0.0025**	**0.0024**	**0.0028**	

Claim – Benefit from Transfer of So2

Description / Location	Unit	Jun-08	Jul-08	Aug-08	Sep-08	Oct-08	Nov-08	Dec-08	Total
Allocation of Saved So2 Emissions based on Monthly Distribution of So2 Emissions:									
Alternative Location 1	*Tonnes*	13	118	106	64	60	70	52	483
Alternative Location 2	*Tonnes*	1	27	34	23	40	69	55	249
Alternative Location 3	*Tonnes*	12	53	84	73	77	73	113	483
Total	***Tonnes***	**25**	**198**	**224**	**160**	**176**	**211**	**219**	**1,214**
Equivalent Generation:									
Alternative Location 1	*Mwh*	3,910	26,020	69,088	33,464	30,075	43,778	30,525	236,860
Alternative Location 2	*Mwh*	192	9,574	12,262	7,886	17,543	23,848	18,937	90,242
Alternative Location 3	*Mwh*	3,611	13,159	21,281	21,955	24,269	19,596	29,221	133,092
Total	***Mwh***	**7,712**	**48,753**	**102,630**	**63,305**	**71,887**	**87,223**	**78,682**	**460,193**
Spark Spread:									
Alternative Location 1	*EUR / Mwh*	0.59	6.66	13.10	19.07	18.61	14.44	9.09	
Alternative Location 2	*EUR / Mwh*	(4.06)	1.85	20.96	5.95	22.72	21.97	12.89	
Alternative Location 3	*EUR / Mwh*	(9.82)	(3.64)	1.14	15.71	23.77	23.12	6.52	
Makeup:									
Alternative Location 1	*EUR*	2,301	173,264	905,296	638,038	559,653	632,022	277,583	3,188,157
Alternative Location 2	*EUR*	(780)	17,744	257,031	46,900	398,552	523,824	244,125	1,487,396
Alternative Location 3	*EUR*	(35,468)	(47,903)	24,230	344,864	576,879	453,093	190,660	1,506,355
Total	***EUR***	**(33,947)**	**143,105**	**1,186,557**	**1,029,803**	**1,535,084**	**1,608,938**	**712,368**	**6,181,908**

Measure

Description / Location	Reference	Unit	Aug-08	Sep-08	Oct-08	Nov-08	Dec-08	Total
Saved So2 Emissions		*Tonnes*	224	160	176	211	219	
Rank:								
CCO2	*Sch B*		8	8	7	5	8	
CCO3	*Sch B*		1	3	3	1	2	
COM4	*Sch B*		2	1	1	3	4	
COM5	*Sch B*		3	2	2	2	3	
Litoral (Unit 1 Only)	*Sch B*		7	4	4	4	1	
TER1	*Sch B*		5	6	6	8	7	
TER2	*Sch B*		6	7	8	7	5	
TER3	*Sch B*		4	5	5	6	6	
Net Generation:								
CCO2	*Sch D*	*Mwh*	2,637	0	0	0	0	2,637
CCO3	*Sch D*	*Mwh*	174,395	211,175	166,225	196,646	169,899	918,339
COM4	*Sch D*	*Mwh*	174,929	118,932	166,271	178,909	158,390	797,432
COM5	*Sch D*	*Mwh*	178,743	146,711	198,302	183,641	183,693	891,090
Litoral (Unit 1 Only)	*Sch D*	*Mwh*	94,188	112,367	309,631	304,619	317,621	1,138,426
TER1	*Sch D*	*Mwh*	47,246	147,609	94,582	63,184	159,240	511,860
TER2	*Sch D*	*Mwh*	48,652	65,745	169,541	19,429	168,099	471,466
TER3	*Sch D*	*Mwh*	67,572	99,471	164,209	167,699	162,773	661,723
Total		***Mwh***	**788,363**	**902,009**	**1,268,760**	**1,114,128**	**1,319,714**	**5,392,974**

Measure

Description / Location	Reference	Unit	Aug-08	Sep-08	Oct-08	Nov-08	Dec-08	Total
Manual Allocation of Equivalent Generation:								
CCO2		*Mwh*	0	0	0	0	0	0
CCO3		*Mwh*	83,590	0	0	134,109	0	217,698
COM4		*Mwh*	0	115,996	118,223	0	0	234,219
COM5		*Mwh*	0	0	0	0	0	0
Litoral (Unit 1 Only)		*Mwh*	0	0	0	0	75,696	75,696
TER1		*Mwh*	0	0	0	0	0	0
TER2		*Mwh*	0	0	0	0	0	0
TER3		*Mwh*	0	0	0	0	0	0
Total			**83,590**	**115,996**	**118,223**	**134,109**	**75,696**	**527,613**
Spark Spread:								
CCO2		*EUR / Mwh*	13.10	19.07	18.61	14.44	9.09	
CCO3		*EUR / Mwh*	13.10	19.07	18.61	14.44	9.09	
COM4		*EUR / Mwh*	13.10	19.07	18.61	14.44	9.09	
COM5		*EUR / Mwh*	13.10	19.07	18.61	14.44	9.09	
Litoral (Unit 1 Only)		*EUR / Mwh*	20.96	5.95	22.72	21.97	12.89	`
TER1		*EUR / Mwh*	1.14	15.71	23.77	23.12	6.52	
TER2		*EUR / Mwh*	1.14	15.71	23.77	23.12	6.52	
TER3		*EUR / Mwh*	1.14	15.71	23.77	23.12	6.52	
Makeup:								
CCO2		*EUR*	0	0	0	0	0	0
CCO3		*EUR*	1,095,318	0	0	1,936,111	0	3,031,429
COM4		*EUR*	0	2,211,608	2,199,987	0	0	4,411,595
COM5		*EUR*	0	0	0	0	0	0
Litoral (Unit 1 Only)		*EUR*	0	0	0	0	975,844	975,844
TER1		*EUR*	0	0	0	0	0	0
TER2		*EUR*	0	0	0	0	0	0
TER3		*EUR*	0	0	0	0	0	0
Total			**1,095,318**	**2,211,608**	**2,199,987**	**1,936,111**	**975,844**	**8,418,868**

Measure

- Established the pre loss historical maximum generation of each plant (Jan 2007-May 2008), the actual generation was then deducted from the historical maximums to establish the amount of generation that was realistically available at each plant.
- The saved So2 emissions were converted into equivalent generation (Mwh) by using the efficiency rates of each plant.
- If the equivalent generation exceeded available generation then the equivalent generation amount was capped at the available amount ‘available equivalent generation’
- Available equivalent generation was then converted into tonnes and Euros using the established efficiency rates and spark spreads respectively. This showed the maximum tonnage that each plant could emit and maximum credit that each plant could obtain based on this.
- In order to rank the plants effectively to allocate the saved So2 emissions to the optimal plants, a Euro per Tonne amount was calculated and then the plants were ranked accordingly.
- If the saved So2 emission exceeded the 1st optimal plants capacity then the remaining tonnes were allocate to the plant ranked next in line. Process continued until all of the saved emissions were allocated.
- This totaled an overall So2 credit amount of **EUR 8.4 million**

Summary

- Decision:
  - How much / if to transfer of So2 emissions to other locations
- Constraints:
  - Limited amount of So2 emissions possible - exceed allowance and face serious penalties
  - Cost of generation at other plants
  - Limited amount of available hours and other plants
- Objective:
  - Maximise generation from other plants whilst not exceed So2 allowance

Conclusion

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